Lesson 9: Work & Energy

1 Work & Energy

In the previous lessons, the motion of a particle was found by solving the equations of motion given by $\vec{F} = m\vec{a}$. If the force acting on a particle was given, the acceleration was then solved for. From the acceleration we integrated to find the velocity and position of the particle.

We will now introduce the principle of work and energy. The advantage of this method is that we will not need to determine the acceleration. Instead the forces will be directly related to the velocity and displacement of the particle.

We define the work of a force F acting on a particle undergoing a displacement $d\vec{r}$ as

$$\boxed{dU = \vec{F} \cdot d\vec{r}}$$

If we know the angle α between the force and displacement vector the work is

$$dU = |\vec{F}|\,|d\vec{r}|\cos\alpha$$

In rectangular components we can express the work as

$$dU = F_x dx + F_y dy$$

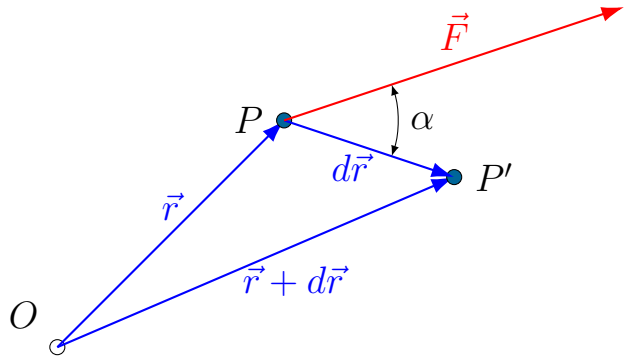


Figure 1: The work done by the force $\vec{F}$ acting on a the particle P having displacement $d\vec{r}$ is the dot product of $\vec{F}$ and $d\vec{r}$.

Work is a scalar quantity having a magnitude but no direction. The dimensions of work are force times length. In the US system it is expressed as ft·lb or in·lb. In the SI system, work is expressed as N·m and is called a joule (J).

The work of a force over a finite displacement is obtained by integrating the above expressions

$$\boxed{U_{1\to 2} = \int_{P_1}^{P_2} \vec{F} \cdot d\vec{r}}$$

This expression can be used to derive formulas for the work done by a force in several common scenarios. We skip the derivations and list the results.

1.1 Work of a constant force in rectilinear motion

$$U_{1\to 2} = (F\cos\alpha)\,\Delta x$$

Sample Problem 9.1

A groundskeeper pushes a lawnmower across a flat lawn, exerting a constant force of 48 Newtons over a distance of 7.5 meters. The handle of the lawnmower is angled such that the applied force is directed at 32° down from horizontal. Calculate the total work done on the lawnmower.

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305 Joules

1.2 Work of the force of gravity

$$U_{1\to 2} = -W\Delta y$$

Sample Problem 9.2

A 75.0-kg person climbs stairs, gaining 2.50 m in height. Find the work done to accomplish this task.

1,837.5 Joules

1.3 Work of the force exerted by a spring

$$U_{1\to 2} = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2$$

Sample Problem 9.3

A perfectly elastic spring requires 0.54 J of work to stretch 6 cm from its equilibrium position. (a) What is its spring constant k? (b) How much work is required to stretch it an additional 6 cm?

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3 N/cm 1.62 J

2 Principle of work and energy

First, let us define what is know as the kinetic energy of a particle

$$\mathrm{KE} = \frac{1}{2}mv^2$$

From newton's 2nd law we can derive the following principle of work and energy:

$$\boxed{\mathrm{KE}_1 + U_{1\to 2} = \mathrm{KE}_2} \tag{1}$$

where KE_1 is the kinetic energy of the particle at position 1. Similarly, KE_2 is the kinetic energy of the particle at position 2. The quantity $U_{1\to 2}$ is the total work of the forces acting on the particle and is found by adding the work done by the individual forces.

Sample Problem 9.4

A 3500 lb car is driven down a 5° incline at a speed of 60 mph when the breaks are applied. The brakes produce a constant total braking force of 1000 lb. (This the force applied by the road onto the tires). Using the principle of work and energy, find the stopping distance of the car.

9.4 – cont

605.6 feet

3 Power and efficiency

Power is defined as the rate in which work is done. Let's say that a certain amount of work ΔU is done over a time interval Δt, then the average power during that time interval is

$$\text{Average Power} = \frac{\Delta U}{\Delta t}$$

The instantaneous power is

$$\text{Power} = \vec{F} \cdot \vec{v}$$

If we use the SI system

$$1 \text{ W} = 1 \text{ J/s}$$

$$1 \text{ hp} = 550 \text{ ft} \cdot \text{lb/s}$$

Sample Problem 9.5

Your car broke down and now needs to be repaired. How much power is required for a lift to raise your 1.2 ton car 6 ft off the ground in 15

9.5 – cont

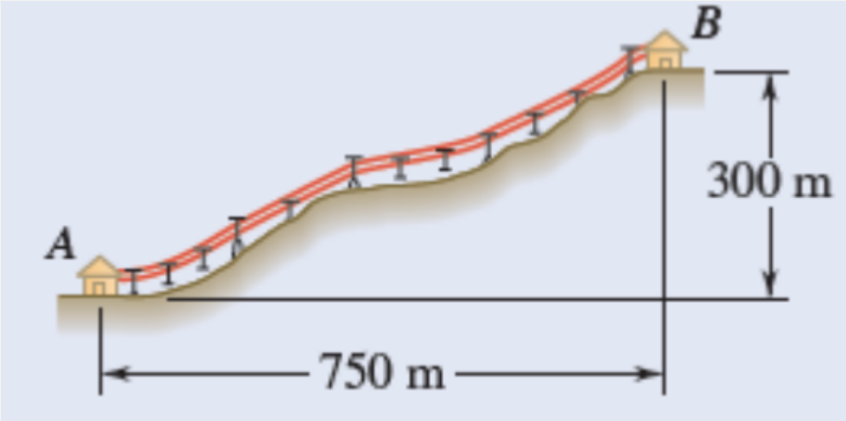
seconds?

$$W = 14,400 \text{ ft} \cdot \text{lb}$$

$$P = 960 \text{ ft} \cdot \text{lb/s} = 1.75 \text{ hp}$$

Sample Problem 9.6

Determine the average power required to operate a chair-lift that is designed to transport 1000 skiers per hour from the base A to the summit B. The average mass of a skier is 70 kg and the average speed of the lift is 75 m/min.

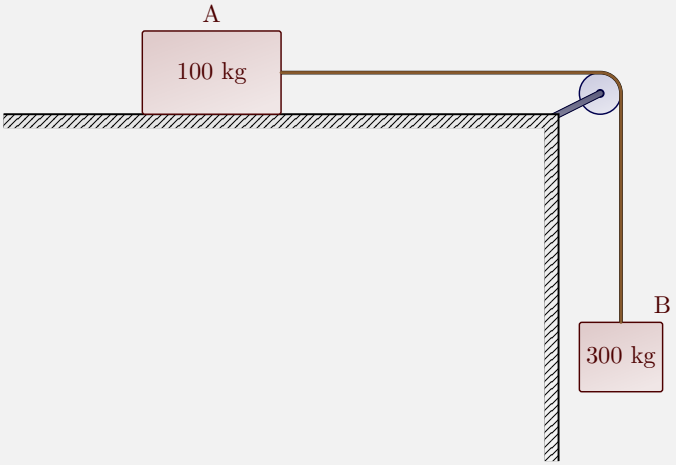


9.6 – cont

57.2 kW

Sample Problem 9.7

Two blocks, A and B are connected by a cable as shown in the figure. The coefficient of friction between block A and the horizontal surface is $\mu_k = 0.2$. The pulley is massless and frictionless.
If the system is released from rest, what is the velocity of block A after it has moved a distance of 2 m? What is the tension in the cable at this instant?



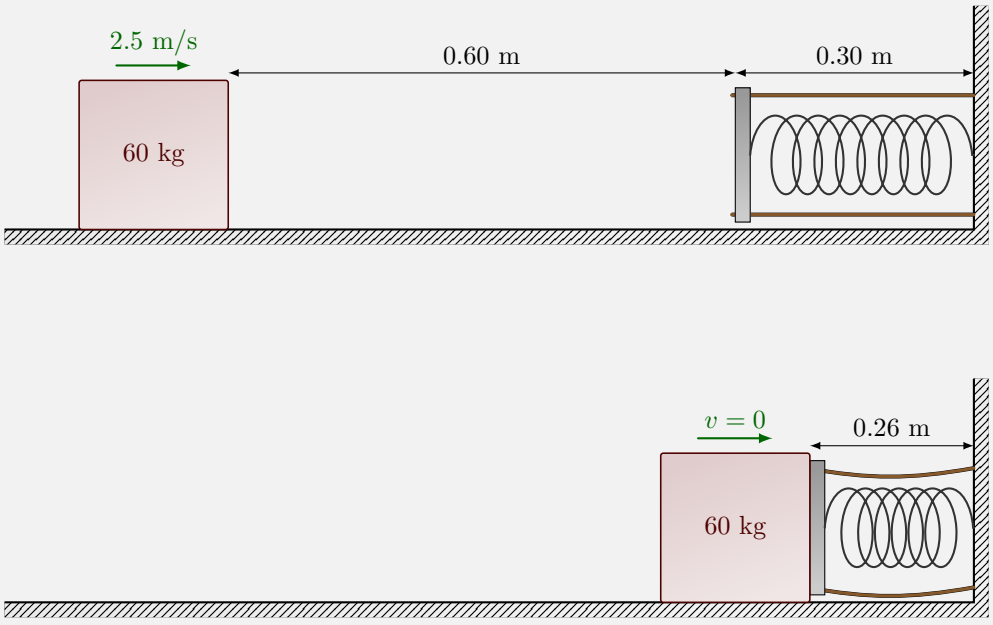
$$v_2 = \sqrt{\frac{2gL(m_B - \mu_k m_A)}{m_A + m_B}} = 5.24$$

$$T = \frac{m_A m_B}{2(m_A + m_B)} gL(1 + \mu_k) = 882.9$$

Sample Problem 9.8

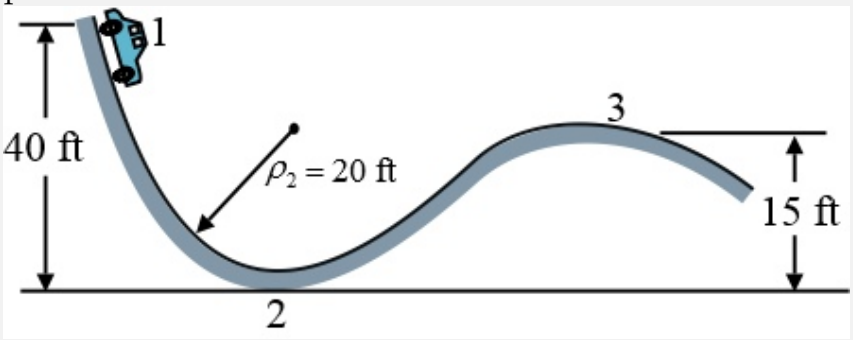
A spring is used to rebound a 60 kg package that is sliding on a horizontal table. The spring constant is $k = 20$ kN/m and is initially pre-compressed by 0.150 m by the restraining cables as shown. (The uncompressed length of the spring is 0.450 m).

The package starts at the location shown in the upper figure with an initial speed of 2.5 m/s. Find the coefficient of friction between the package and the table such that the spring has a maximum additional deformation of 0.04 m as depicted in the lower figure. Then find the final location of the right-edge of the package as measured from the wall.



Sample Problem 9.9

A 2000-lb car starts from rest at point 1 and moves without friction down the track shown.
(a) Determine the force exerted by the track on the car at point 2, where the radius of curvature of the track is 20 ft.
(b) Determine the minimum safe value of the radius of curvature at point 3.



Lesson 9 Sample Problems

1. A groundskeeper pushes a lawnmower across a flat lawn, exerting a constant force of 48 Newtons over a distance of 7.5 meters. The handle of the lawnmower is angled such that the applied force is directed at 32° down from horizontal. Calculate the total work done on the lawnmower.

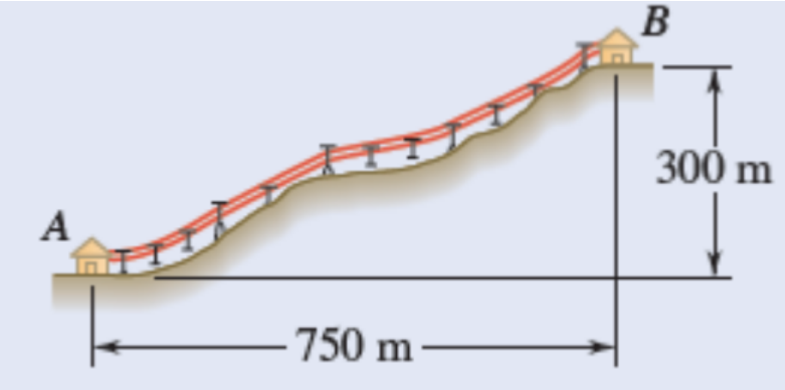
2. A 75.0-kg person climbs stairs, gaining 2.50 m in height. Find the work done to accomplish this task.

3. A perfectly elastic spring requires 0.54 J of work to stretch 6 cm from its equilibrium position. (a) What is its spring constant k? (b) How much work is required to stretch it an additional 6 cm?

4. A 3500 lb car is driven down a 5° incline at a speed of 60 mph when the breaks are applied. The brakes produce a constant total braking force of 1000 lb. (This the force applied by the road onto the tires). Using the principle of work and energy, find the stopping distance of the car.

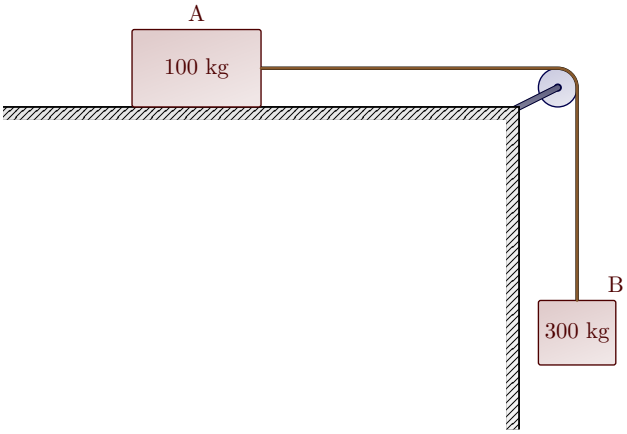
5. Your car broke down and now needs to be repaired. How much power is required for a lift to raise your 1.2 ton car 6 ft off the ground in 15 seconds?

6. Determine the average power required to operate a chair-lift that is designed to transport 1000 skiers per hour from the base A to the summit B. The average mass of a skier is 70 kg and the average speed of the lift is 75 m/min.



7. Two blocks, A and B are connected by a cable as shown in the figure. The coefficient of friction between block A and the horizontal surface is $\mu_k = 0.2$. The pulley is massless and frictionless.

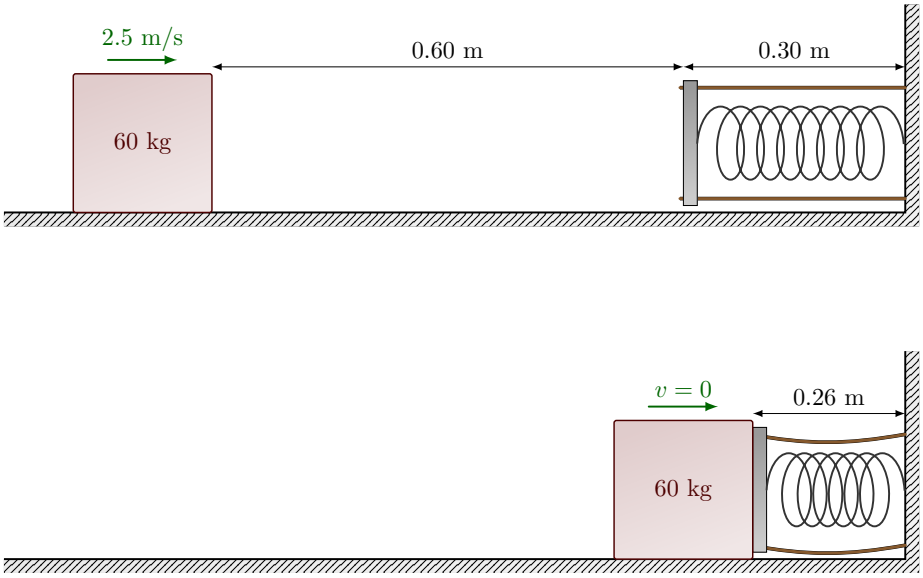
   If the system is released from rest, what is the velocity of block A after it has moved a distance of 2 m? What is the tension in the cable at this instant?



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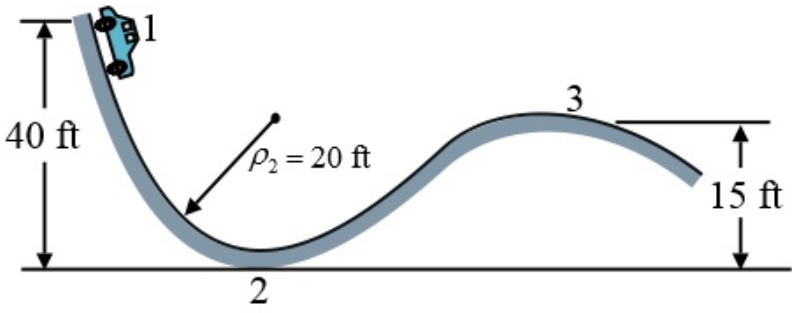
with an initial speed of 2.5 m/s. Find the coefficient of friction between the package and the table such that the spring has a maximum additional deformation of 0.04 m as depicted in the lower figure. Then find the final location of the right-edge of the package as measured from the wall.



9. A 2000-lb car starts from rest at point 1 and moves without friction down the track shown.

   (a) Determine the force exerted by the track on the car at point 2, where the radius of curvature of the track is 20 ft.

   (b) Determine the minimum safe value of the radius of curvature at point 3.


1
40 ft
$\rho_2 = 20$ ft
3
15 ft
2